

PHASE PROPERTIES OF MULTICOMPONENT SUPERPOSITION STATES IN VARIOUS AMPLIFIERS

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Abstract

There have been theoretical studies for generation of optical coherent superposition states. Once the superposition state is generated it is natural to ask if it is possible to amplify it without losing the nonclassical properties of the field state. We consider amplification of the superposition state in various amplifiers such as a sub-Poissonian amplifier, a phase-sensitive amplifier and a classical amplifier. We show the evolution of phase probability distribution functions in the amplifier.

1 INTRODUCTION

The superposition principle lies at the heart of quantum mechanics according to Dirac [1]. In this paper we consider the amplification of optical superposition states. As a result of interaction of a single mode coherent state with a nonlinear Kerr medium, the coherent state input becomes a generalized coherent state [2 - 3]. The dynamics of a single mode field propagating in the Kerr medium is governed by the effective Hamiltonian [2] $\hat{H} = \omega\hat{n} + \lambda\hat{n}^2$, where λ is the non-linear factor and \hat{n} is the photon number operator. Under the influence of the nonlinear interaction the initial coherent state $|\alpha\rangle$ of the amplitude α evolves at time t into the state

$$|\Psi(t)\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right) \sum_{k=0}^{\infty} \frac{\alpha^k e^{-i\Phi_k}}{\sqrt{k!}} |k\rangle, \quad \Phi_k = \lambda t k^2, \quad (1)$$

where $|k\rangle$ is a Fock state. At the interaction time $t = \pi/N\lambda$, we can rewrite Eqn(1) as a form

$$|\Psi\rangle = \frac{1}{\sqrt{N}} \sum_{n=1}^N e^{i\zeta_n} |-\alpha e^{2in\pi/N}\rangle, \quad (2)$$

which is a superposition of N coherent component states located on a circle with the centre at the origin of phase space. If we decompose the state (2) into the Fock basis and compare it with Eqn(1) we obtain an equation for the arguments, ζ_n , of the coefficients of coherent components for an arbitrary value of N . For example when $N = 4$ we have

$$|\Psi\rangle = \frac{1}{2} \left(e^{-i\pi/4} |\alpha\rangle - e^{-i\pi/4} |-\alpha\rangle + |i\alpha\rangle + |-i\alpha\rangle \right), \quad (3)$$

which we call the Yurke-Stoler state throughout the paper.

Such superpositions of multicomponent coherent states may be generated not only in the amplitude dispersive medium but also in the micromaser type experiment. Recently Garraway et al. have proposed a method to prepare quantum superposition of multicomponent states [4, 5]

for the eventual goal of preparation of a Fock state. A stream of three-level atoms are injected into a high-Q micromaser cavity. It is assumed that there is just one atom at a time in the cavity and the initial coherent cavity field is tuned to the two-photon resonance with the atomic transition. A superposition state of two coherent component states is generated by a conditional measurement of the atomic excitation after an interaction time that determines the relative phase of the component states. By a sequence of the conditional measurements the superposition of multicomponent coherent states may be generated. As a special case, the initial interaction time is chosen to create a superposition state of two component states separated in phase space by π and the interaction times are reduced by one half after each interaction. The second conditional measurement creates a superposition of four component states

$$|\Psi\rangle = \frac{1}{\sqrt{\aleph}} \left[e^{i\frac{3}{4}\pi} |\alpha e^{i\frac{3}{4}\pi}\rangle + e^{-i\frac{3}{4}\pi} |\alpha e^{-i\frac{3}{4}\pi}\rangle + e^{i\frac{3}{4}\pi} |\alpha e^{i\frac{1}{4}\pi}\rangle + e^{-i\frac{3}{4}\pi} |\alpha e^{-i\frac{1}{4}\pi}\rangle \right], \quad (4)$$

where \aleph is a normalization constant. Eventually, the N th measurement creates the superposition of 2^N components separated by $2\pi/2^N$ in phase space. We call the state (4) as the Garraway state.

2 PHASE PROBABILITY DISTRIBUTION FUNCTION

There are quasiprobability distributions according to ordering of system operators. One is the Glauber P representation which is quasi probability distribution function for the normal ordering of the system operators and another is the Q function $Q(re^{i\theta})$ for antinormal ordering of system operators [6]. The Wigner function $W(re^{i\theta})$ for the symmetrical ordering can be negative. Since the P function cannot be defined in the nonclassical regime, the P function is not dealt with in this paper.

We can study the phase probability of the system with help of the quasiprobabilities. We derive two phase probability distributions from the Q and Wigner quasiprobability distributions [7]:

$$P^{(W)}(\theta) = \int_0^\infty r W(re^{i\theta}) dr \quad (5a)$$

$$P^{(Q)}(\theta) = \int_0^\infty r Q(re^{i\theta}) dr \quad (5b)$$

Buzek et al. [7] compare these two probability distributions with phase probability defined by Pegg and Barnett [8]. As for the Wigner function, the Wigner phase probability distribution can have negative values, which indicate nonclassical nature of the system. For the quantum superposition state, the quantum interference between component states are best illustrated by the Wigner phase distribution as it can become negative for the quantum interference of component states. As shown in Fig.1a the quantum interference for the Yurke-Stoler state is reflected by the negative values of the Wigner phase probability function. However the Wigner function for the Garraway state is always positive as in Fig.1b. We can therefore conclude that the quantum interference is not necessarily represented by negative values in the Wigner phase distribution. The Q phase distribution function is always positive differently from the Wigner phase distribution function as shown in Fig.1.

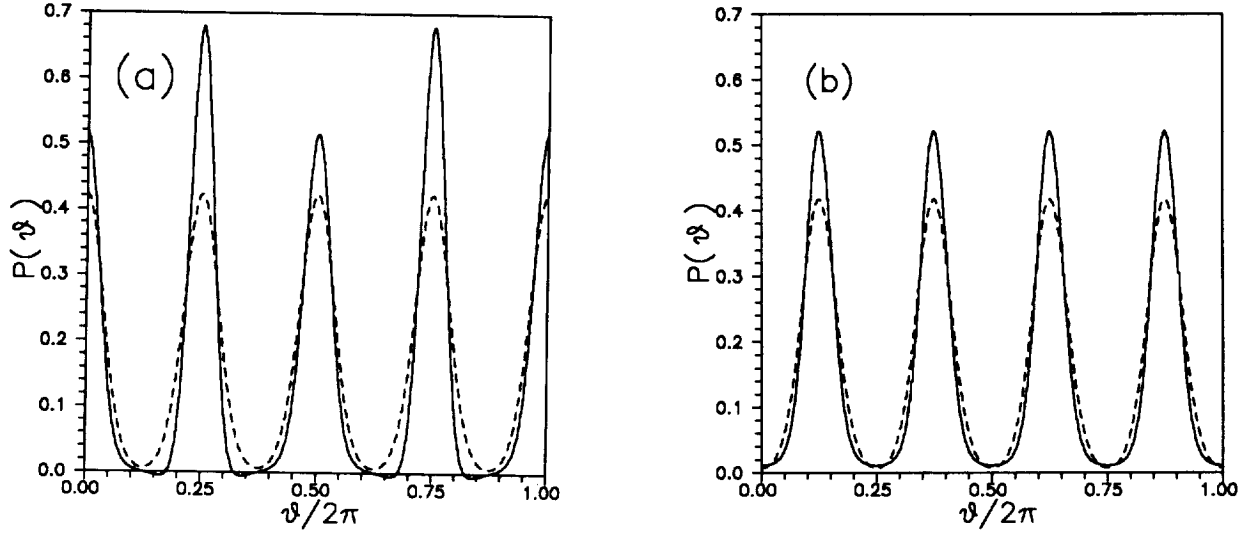


Fig.1. Wigner and Q phase distribution of the Yurke-Stoler (a) and Garraway (b) state. Solid line shows the Wigner phase distribution and dashed line shows the Q phase distribution. ($\alpha = 3$)

3 AMPLIFIED SUPERPOSITION STATES

The simplest way to amplify the state of the single-mode field is to displace it by the displacement operator $\hat{D}(\sigma)$ [6]. This operator shifts a field state by a given amplitude in phase space. The displacement of a field state can be implemented by driving the field by a classical current. The displaced superposition state is expressed by

$$|\Psi\rangle = \hat{D}(\sigma)|\psi_0\rangle, \quad \hat{D} = \exp\left(\sigma\hat{a}^\dagger - \sigma^*\hat{a}\right), \quad (6)$$

where $|\psi_0\rangle$ is the initial superposition state. From definitions of the quasiprobability distributions it follows that their shapes are invariant with respect to the action of the displacement operator. The only difference consists in the shift of the Wigner function of the state in phase space along the action of the displacement operator. One of consequences of this invariance is that the displacement of the state does not change the quadrature-squeezing properties associated with the original state. As the whole picture is displaced along one direction in phase space the phase distribution will be differed by the action of displacement as shown in Fig.2. The Wigner phase distribution becomes to have negative values by displacement.

Recently the correlated (phase-sensitive) amplifier has been realized experimentally [9]. In this section we study the evolution of the phase probability distribution of the superposition states amplified by the phase-sensitive amplifier. The dynamics of the field mode coupled to the phase-sensitive amplifier is in the Born and Markov approximation governed by the Fokker-Planck equation for the Q function, which in the interaction picture can be written as [10]

$$\frac{\partial Q(\alpha, t)}{\partial t} = \gamma \left[N_0 \frac{\partial^2}{\partial \alpha^* \partial \alpha} - \frac{1}{2} \left(\frac{\partial}{\partial \alpha^*} \alpha^* + \frac{\partial}{\partial \alpha} \alpha \right) + \frac{M_0^*}{2} \frac{\partial^2}{\partial \alpha^2} + \frac{M_0}{2} \frac{\partial^2}{\partial \alpha^{*2}} \right] Q(\alpha, t), \quad (7)$$

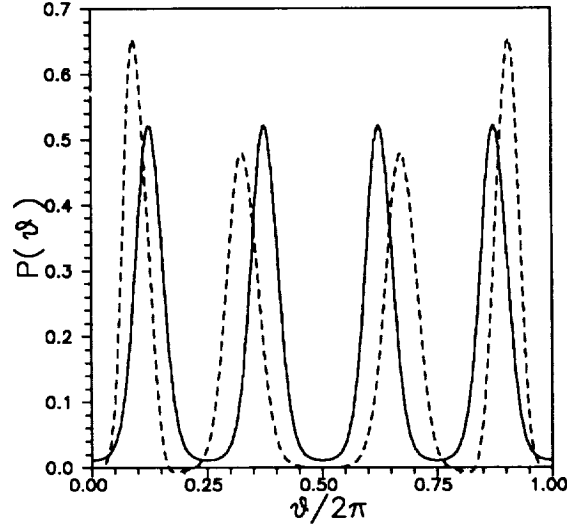


Fig.2. Wigner phase distribution for the displaced Garraway superposition state. Solid line is for the initial state and dashed line is for the displaced state ($\sigma = 1, \alpha = 3$).

where γ is proportional to the coupling between the field mode and the environment, and N_0 and M_0 are the parameters determined by nature of the amplifier. If the phase-sensitive parameter M_0 is equal to zero then the Fokker-Planck equation (8) reduces into the equation describing the phase-insensitive amplification of the single mode field. The gain G of the amplifier is defined as $G = \exp(\gamma t)$.

Fig.3 clearly shows that the choice of the M value determines into which quadrature noise should be added. When M is positive the phase information at $\vartheta = 0$ and π is kept while the phase information at $\vartheta = \pi/2$ and $3\pi/2$ is lost very rapidly. However, when M is negative the information at $\vartheta = \pi/2, 3\pi/2$ is kept at the expense of the rapid loss of the information at the other quadrature.

We consider a stream of atoms injected into a micromaser cavity with an infinite Q . We assume that there is just one atom at a time in the cavity and the atom makes the two-photon transition of frequency ω_0 between the nondegenerate ground and excited states via a single intermediate level. The cavity is assumed to be tuned to the two-photon resonance with the excited and ground levels and the intermediate level is so detuned not to be excited, so that one photon transitions can be neglected which means that the transition between the ground and excited levels can be considered as a two-photon process.

Let us assume that the field mode is initially prepared in a pure state

$$|\Psi(0)\rangle = \sum_{k=0}^{\infty} C_0(k)|k\rangle. \quad (8)$$

and the atom is prepared in the excited state. During the time evolution the atom and the field become strongly entangled [4]. Nevertheless, at some particular moments they become dynamically disentangled and are again in their pure states. One of those moments is identical to the revival

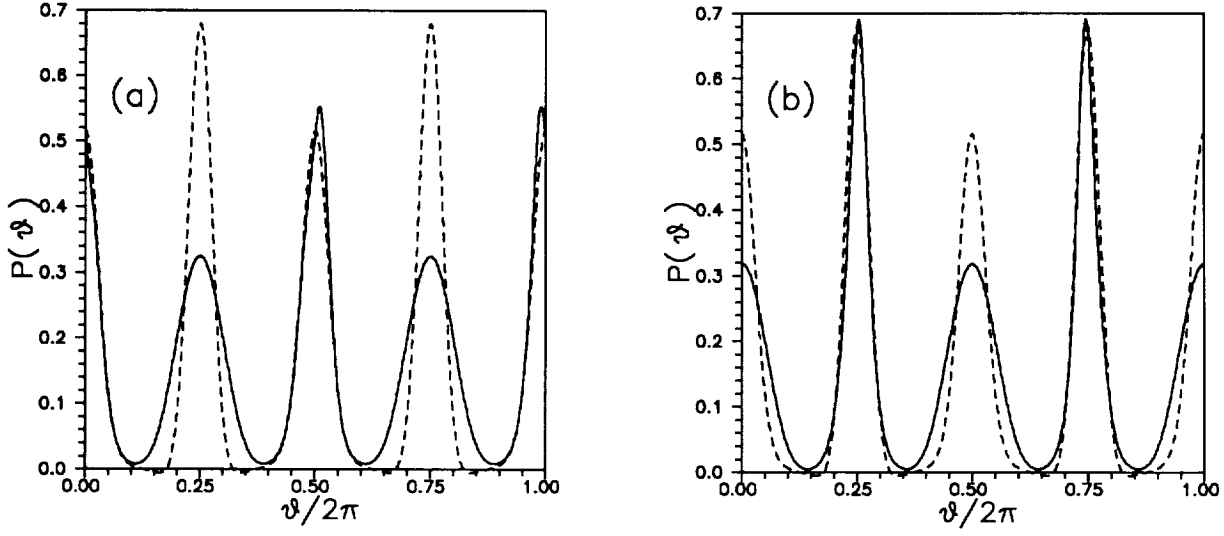


Fig.3. Wigner phase distribution for the Yurke-Stoler state in the phase sensitive amplifier characterized by $N_0 = 3, M_0 = \sqrt{12}$ (a) and by $N_0 = 3, M_0 = -\sqrt{12}$ (b). Dashed line is for the initial state and solid line is for the state at $G = 1.22$ ($\alpha = 3$)

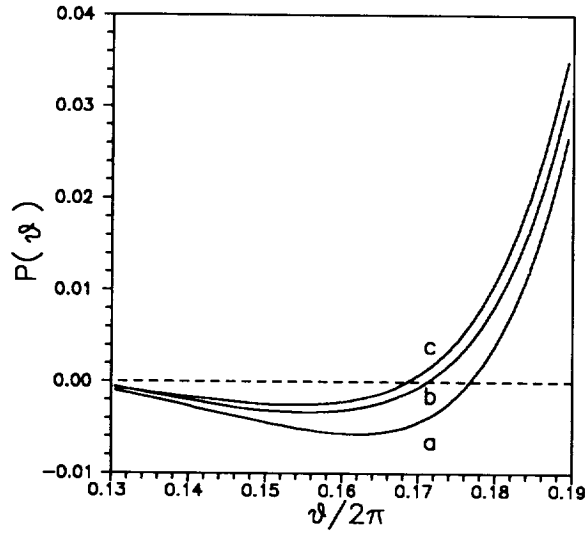


Fig.4. Part of the Wigner phase distribution of the Yurke-Stoler state in the sub-Poissonian amplifier. Line a is for the initial state, line b is for the state with two extra photons and line c is for the state with four extra photons. ($\alpha = 3$)

time t_R (see ref.[4] for details) when the atom is (approximately) in its ground state and the field can be described by the state vector

$$|\Psi(1)\rangle = \sum_{k=0}^{\infty} C_0(k)|k+2\rangle. \quad (9)$$

At this moment exactly two photons are transferred from the atom to the field, so the mean photon number of the field is $\bar{n}(1) = \bar{n}(0) + 2$, where $\bar{n}(i)$ is the mean photon number of the cavity field after i atoms pass the cavity. Analogously, after a sequence of M atoms each of which interacts for time t_R with the cavity field, the state vector of the field can be written as:

$$|\Psi(M)\rangle = \sum_{k=0}^{\infty} C_0(k)|k+2M\rangle. \quad (10)$$

We call the process during which the exact number of photons are transferred to the field as the sub-Poissonian (amplitude-squeezed) amplification.

In Fig.4 we can see that the Wigner phase distribution function is smoothened by the amplification. This means that the phase uncertainty is enlarged as the number uncertainty (so that the energy uncertainty) is reduced by the sub-Poissonian amplification.

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